# Relations and Functions

## Functions

#### Definition:

* A relation from to is a **function** from to IFF:

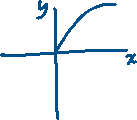
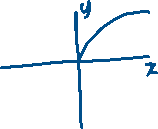
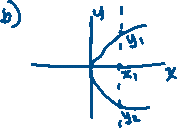
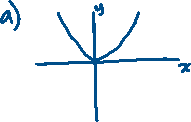
1. , and
2. For each there is at most **one** such that .
   1. Then is the **codomain** of .

* A function from to is denoted by .
* The equation means .
* In that case, is the **image** of under .
* Relations on can be plotted by drawing all the points.
* Such relations are functions if they satisfy the **vertical line test**
* Every vertical line cuts the graph at most once.

Exercise:

Sketch the relations and determine which are functions.

1. On ,
2. On
3. On
4. On



Exercise:

Which are functions?

1. The identity relation on .



1. On



1. On



### Definition (Injective):

* Let be a function.
* We say that is **one-to-one (injective)** IFF for all .
* That is, each element of the range is the image of only **one** element of the domain.

Exercise:

Let , is the number of elements in . Prove or disprove that is one-to-one.



Exercise:

Which are one-to-one?

1. On



1. On



1. On



1. On



### Definition (Surjective):

* A function is **onto (surjective)** IFF .
* That is, for all , there exists such that .

Exercise:

Let . Which are onto?



### Theorem (Inverse):

* The **inverse** of a function , written , is also a function IFF is one-to-one and onto **(bijective)**.

Exercise:

Sketch . Find and sketch . Is a function?

